

## Book Review

Yaroslav D. Sergeyev (2003), *Arithmetic of infinity*, Edizioni Orizzonti Meridionali, 112 pp., ISBN 88-89064-01-3.

This small book deals two concepts that are among the most fundamental mathematics and in general in all natural sciences: concept of number and concept of infinity. The goal of the book is to construct a type of arithmetic that would allow us to introduce and to treat infinite and infinitesimal numbers in the same planner as we are used to do with finite ones, i.e., by applying the philosophical principle of Ancient Greeks ‘the part is less than the whole’.

The problem of infinite and infinitesimal in mathematics, physics, and philosophy has attracted the attention of the most brilliant thinkers in the history of humanity from Aristotle and Zeno to Hilbert and Gödel. For example, much of the earliest development of the infinitesimal calculus was formulated, by Newton and Leibniz. On the other hand, problem of infinity has been deeply studied by Cantor, Robinson, and Conway. There exists an extensive literature on this subject considering various accepts of the topic. However, the author was able give a significant original contribution to the field.

The novelty of the approach proposed by the author consists of on the one hand, in its strong applied character and on the other hand, in its synergy of analysis and sets theory. In fact, the book proposes a new numeral system allowing us to express finite, infinitesimal, and infinite numbers and to execute arithmetical operations with them in a unique framework. Technically, this is done by extending the set of axioms for real numbers through introduction of a new numeral expressing the totality of elements of the set of natural numbers. The consideration is made from positions of applied mathematics and theory of computations without usage of both non-standard analysis tools and such concepts as one-to-one correspondence, numerability, and continuum.

The introduced viewpoint leads to many new powerful tools in analysis, computer science, set theory, measure theory, etc. for instance, it allows

one to substitute classification of infinite Sets with respect to cardinal numbers by simple procedure for direct calculating the number of elements of infinite sets. It becomes possible to substitute limits by arithmetical expressions and to calculate these expressions in different infinite or infinitesimal points even when limits do not exist or when indeterminate forms appear. Divergent series can be calculated, and divergent processes can be studied at different infinite points. It becomes also possible to calculate the number of elements of infinite sequences.

This incomplete list of new results shows that the author has successfully considered the problem of infinity from positions of applied mathematics and theory of computations – fields being among the main scientific interests of the author (for his results in these fields he has been recently awarded a prestigious full professorship in Italy reserved by the Italian Government for distinguished foreign scientists). The expressed viewpoint on infinity gives possibilities to solve new applied problems using arithmetical operations with infinite and infinitesimal numbers that can be executed in a simple and clear way.

It is important to notice an extremely clear exposition of the material. The excellent didactical style of the author is well known to the readers of his previous fundamental book in global optimization and parallel computations published by Kluwer Academic Publishers. It is also worthwhile to mention that for his didactical skills he was awarded three times by prizes in national competitions in the former Soviet Union. Since also in this book the author was able to combine successfully new deep ideas and a very intuitive presentation, I recommend this book not only to professional mathematicians but also to amateurs.

In conclusion, this is a valuable book for both students and researchers interested in mathematics, physics, computer science, and philosophy. The book opens new horizons in these fields and I strongly recommend it to anyone who at least once asked himself about the nature of infinite and infinitesimal. Readers will find this book to be an excellent source of new powerful ideas and future successful developments.

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